An Introduction to Mechanics of Materials

Mechanics I

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CAL POLY

An Introduction to Mechanics of Materials

Beams: Shear Stresses

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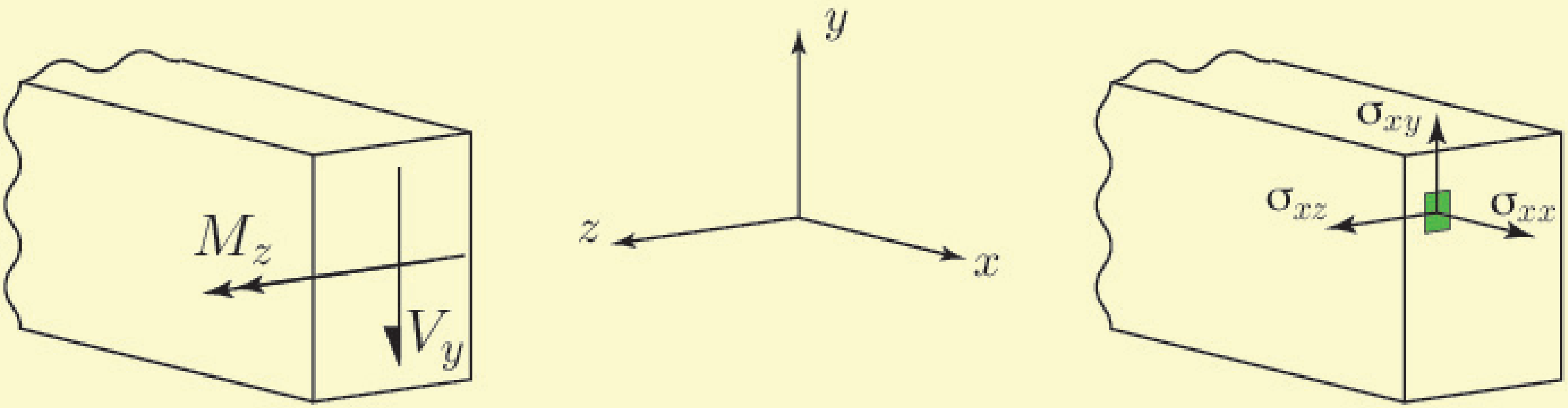
Transverse loading

- **With pure bending the only stress resultant present is a bending moment and thus the only nonzero stress is the longitudinal normal stress**
- **In general most beams are subjected to both internal bending moments and internal shear forces**
- **Internal moments and internal shear lead to normal and shear stresses**
- **The balance law relates the internal resultant forces (M, V) to the stresses (σ)**

- Transverse loading

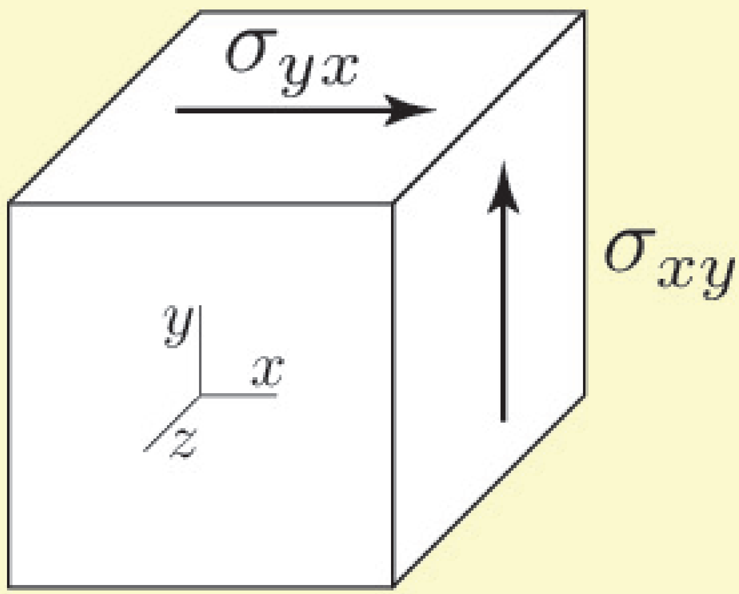


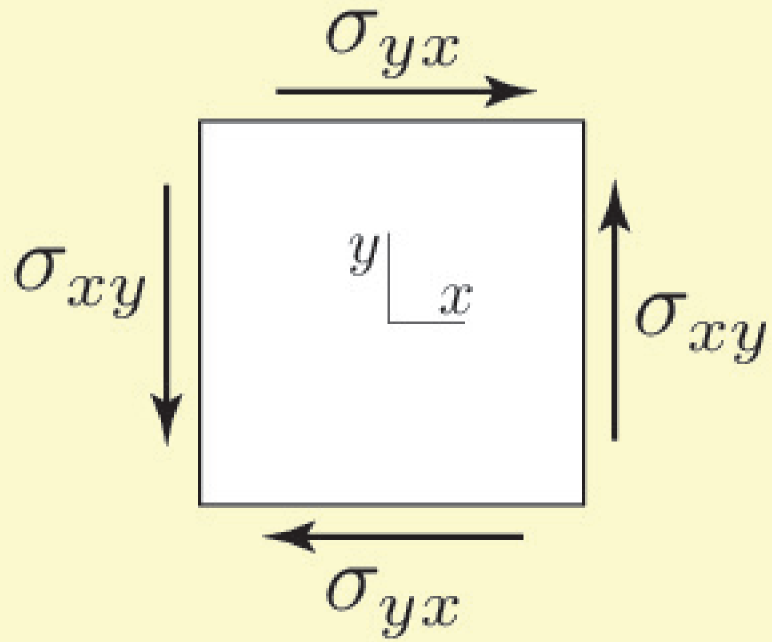
- Balance Law: Resultants ↔ Stresses

$$-V_y = \int \sigma_{xy}\, dA\,; \quad 0 = \int \sigma_{xz}\, dA\,; \quad -M_z = \int y\sigma_{xx}\, dA$$

Beams: Shear Stresses

- Transverse loading

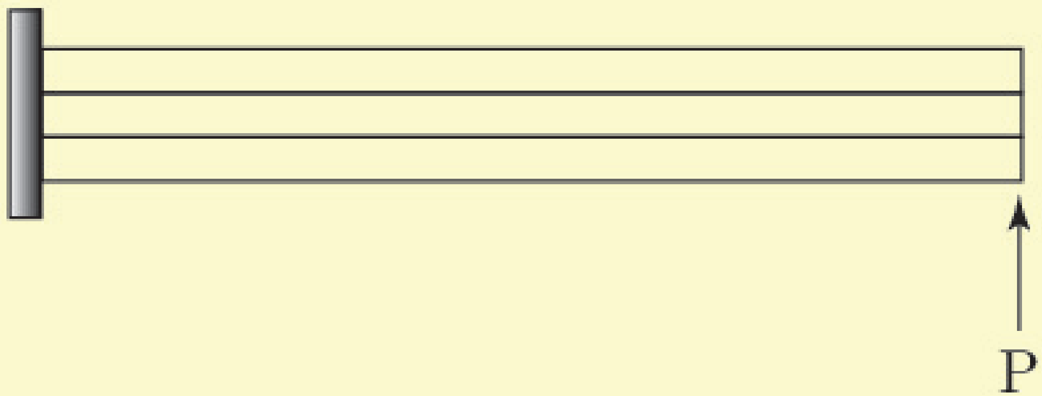


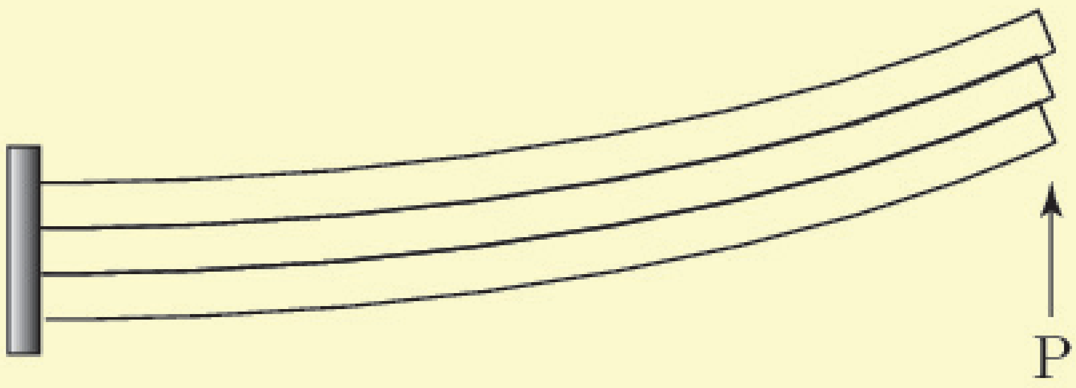


  - Moment equilibrium leads to $\sigma_{xy} = \sigma_{yx}$
  - Similarly $\sigma_{xz} = \sigma_{zx}$ when present in beam

Beams: Shear Stresses

- Transverse loading - illustration





Beams: Shear Stresses

Transverse loading - illustration

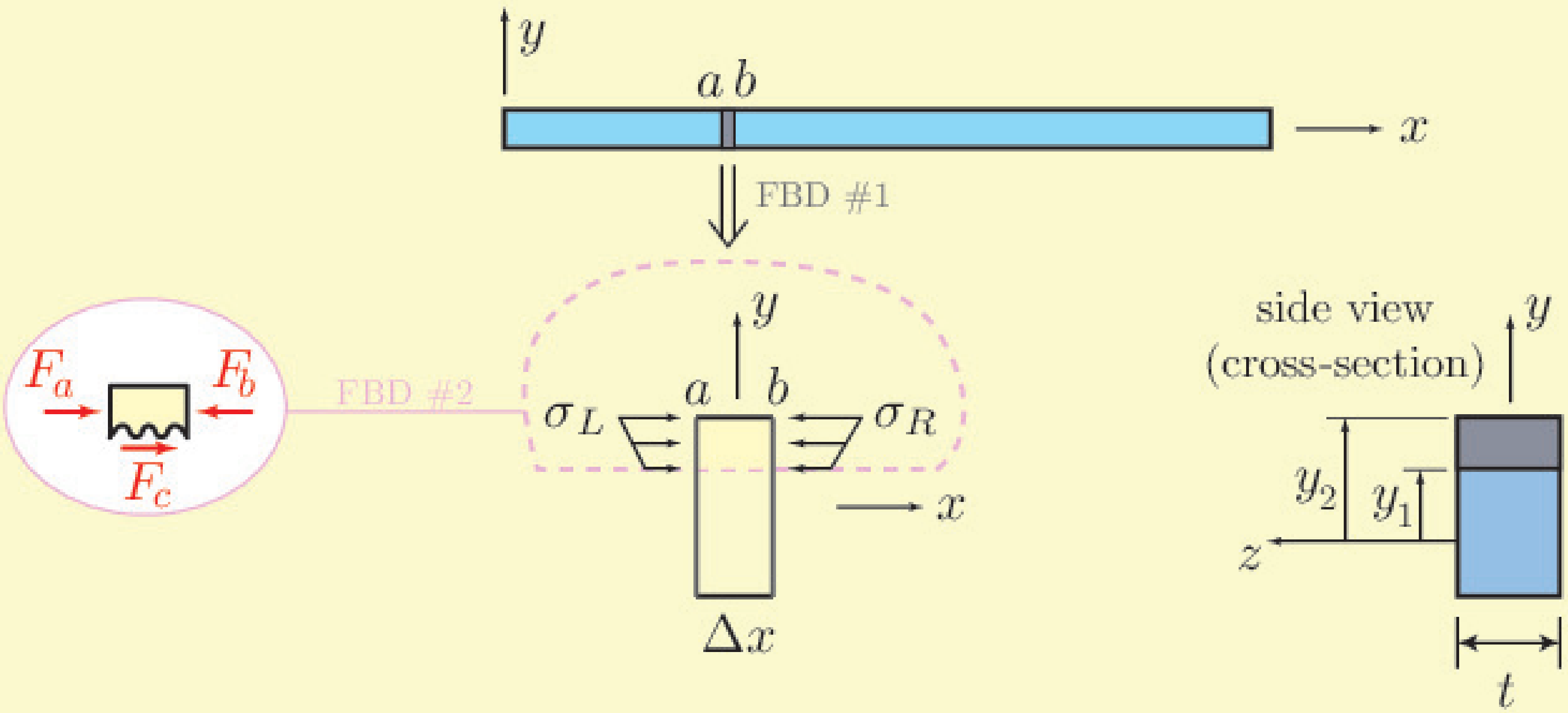
- **The unglued boards slide relative to one another, whereas the "glued" boards bend together as a unit and longitudinal shear flow/stresses are developed in the layers**

- **If the system was subject to a pure bending state the boards would bend into concentric circles and would not slide relative to each, hence no shear flow/stress would exist**

Beams: Shear Stresses

- Horizontal shear flow
  - **Assumptions**
    - Shear strains do not substantially affect the longitudinal strains, hence the flexure formula is justifiable for nonuniform bending
    - The assumptions that were utilized in the development of the flexure formula must also hold here, namely linear elastic material and small deformations/displacements
  - **Remark**
    - If the shear is constant along the longitudinal axis of the beam the longitudinal strain due to the bending moments is unaffected by the shear strain and the distribution of the normal stress is the same as pure bending

Horizontal shear flow

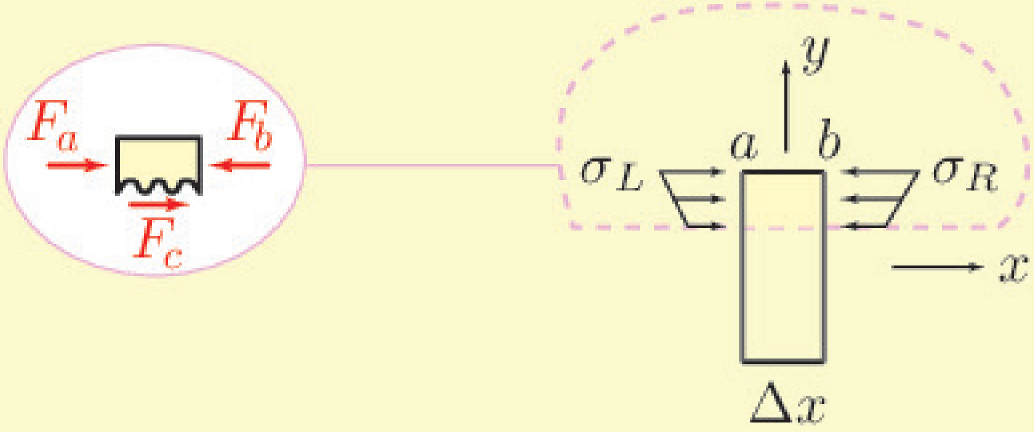


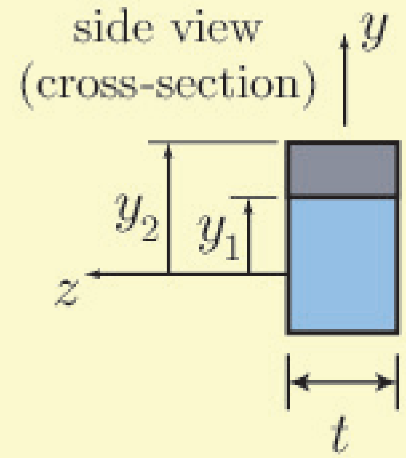
- Horizontal equilibrium

$$\sum F_x = 0 \quad \Rightarrow \quad F_c = F_b - F_a$$

Beams: Shear Stresses

Horizontal shear flow





- Horizontal equilibrium

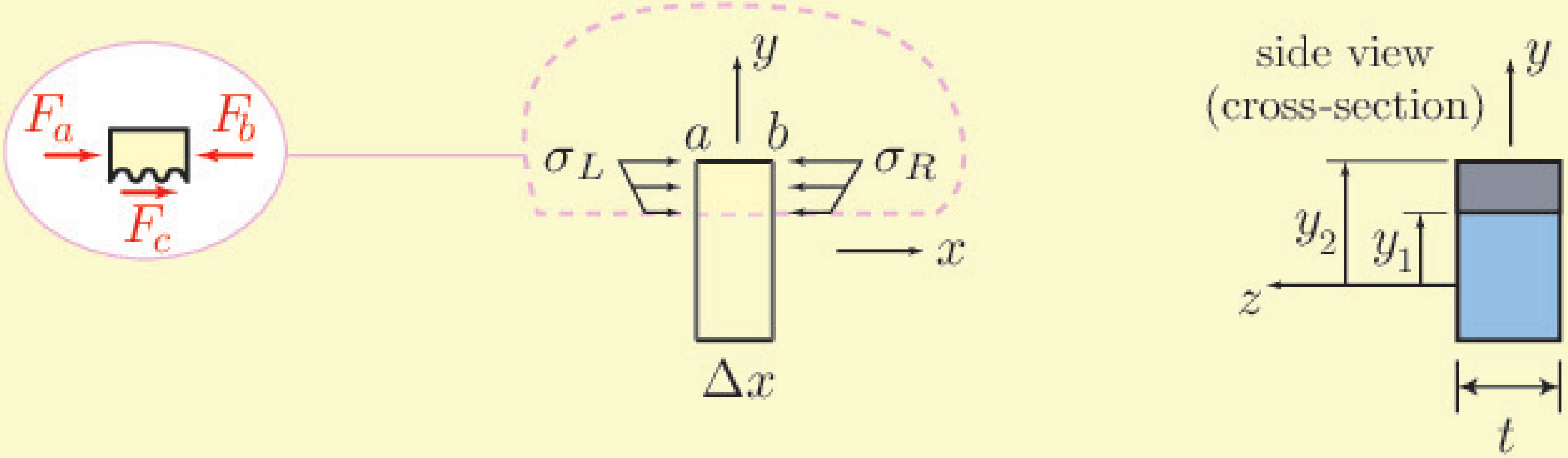
$$F_a = \int \sigma_L \, dA = \int_0^t \int_{y_1}^{y_2} \sigma_L \, dydz = \int_0^t \int_{y_1}^{y_2} \frac{M_L y}{I_{zz}} \, dydz$$

$$= \frac{M_L}{I_{zz}} \int_0^t \int_{y_1}^{y_2} y \, dydz = \frac{M_L Q_z}{I_{zz}}$$

Note that Q_z is the first moment of the shaded area above y_1.

Beams: Shear Stresses

Horizontal shear flow



- Horizontal equilibrium

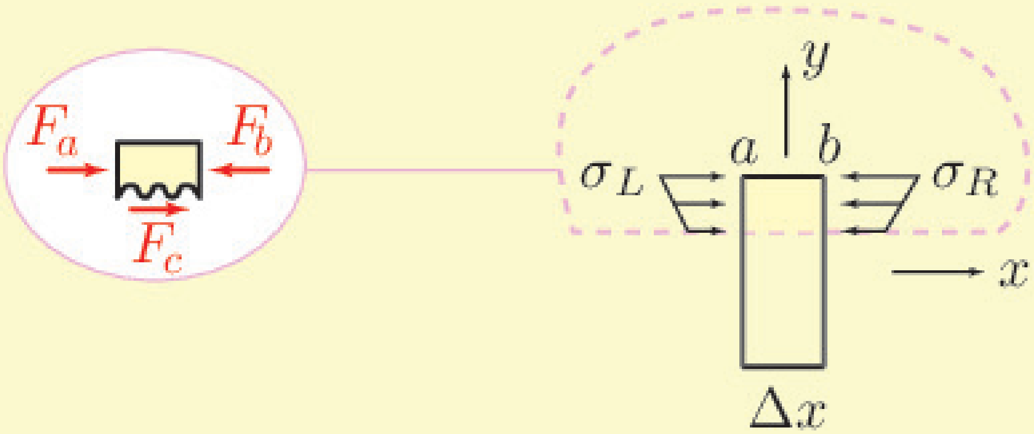
$$F_b = \int \sigma_R \, dA = \int_0^t \int_{y_1}^{y_2} \sigma_R \, dydz = \int_0^t \int_{y_1}^{y_2} \frac{M_R y}{I_{zz}} \, dydz$$

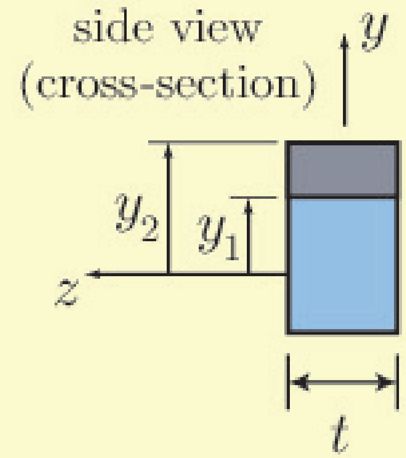
$$= \frac{M_R}{I_{zz}} \int_0^t \int_{y_1}^{y_2} y \, dydz = \frac{M_R Q_z}{I_{zz}}$$

Note that Q_z is the first moment of the shaded area above y_1.

Beams: Shear Stresses

Horizontal shear flow





- Horizontal equilibrium

$$F_c = F_b - F_a = \frac{M_R Q_z}{I_{zz}} - \frac{M_L Q_z}{I_{zz}} = \frac{\Delta M Q_z}{I_{zz}}$$

where ΔM is the change in the moment across the distance Δx.

Beams: Shear Stresses

Horizontal shear flow

- The force per unit length is:

$$\frac{F_c}{\Delta x} = \frac{\Delta M}{\Delta x}\frac{Q_z}{I_{zz}}$$

- Taking the limit as $\Delta x \to 0$ gives:

$$\left(\frac{dM}{dx}\right)\frac{Q_z}{I_{zz}} = \frac{VQ_z}{I_{zz}}$$

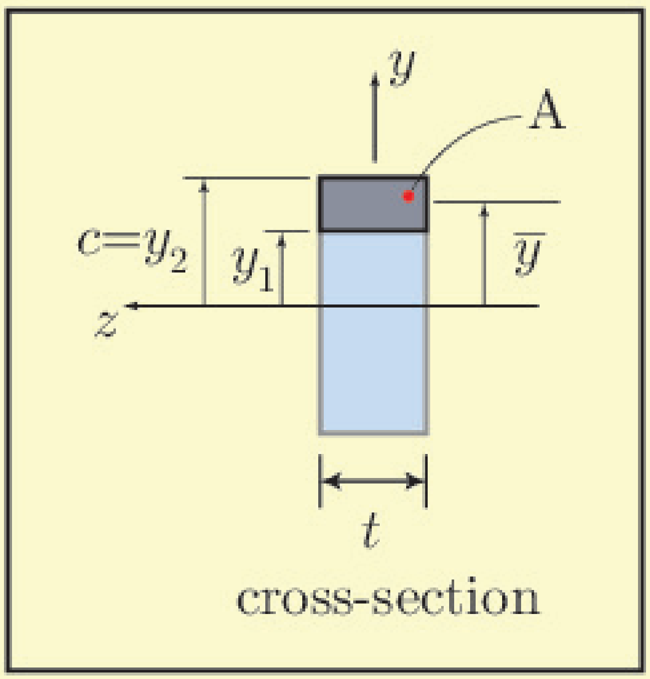
- The result is defined as the *shear flow*:

$$q \equiv \frac{VQ_z}{I_{zz}}$$

Remarks

- Q_z is the first moment of the shaded cross sectional area separated by FBD #2. The choice of FBD #2 implicitly determines where the shear flow q is being evaluated.
- Recall

$$\bar{y} = \frac{Q_z}{A} \quad \Rightarrow \quad Q_z = \bar{y}A$$



Remarks

- Note the first moment of area, Q_z below the level $y = y_1$ is equal in magnitude and opposite in sign to the Q_z located above that line

- The first moment of area, Q_z is a maximum for $y_1 = 0$, since elements of the cross section above the neutral axis contribute positively to the integral

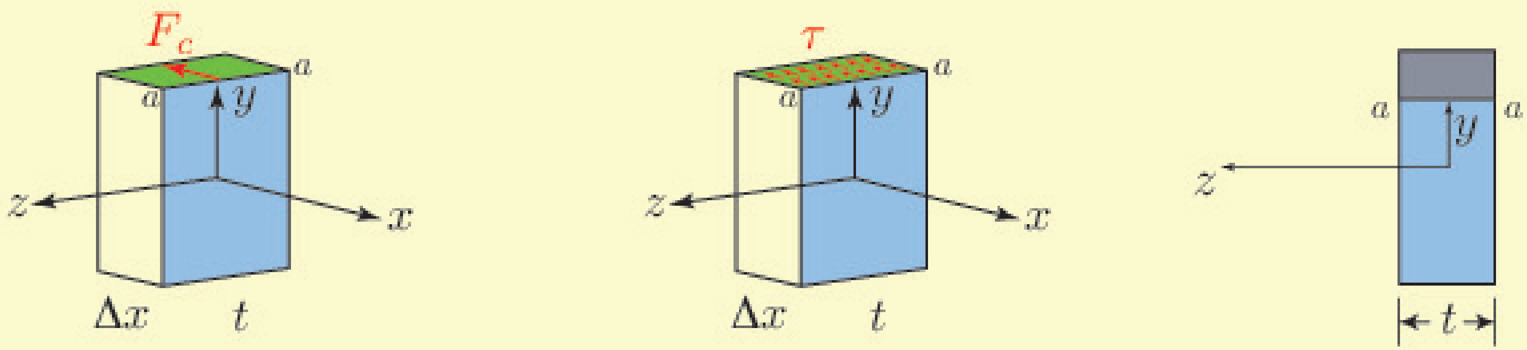
$$Q_z = \int_{y_1}^{y_2} y \, dA$$

  while elements below contribute negatively

- For pure bending the shear $V = 0$, hence the horizontal shear flow is also zero, i.e. $q = 0$

Beams: Shear Stresses

Shear stress



- The average shear stress is:

$$\tau_{avg} = \left[\frac{F_c}{t\Delta x}\right]_{\Delta x \to 0}$$

- The result is:

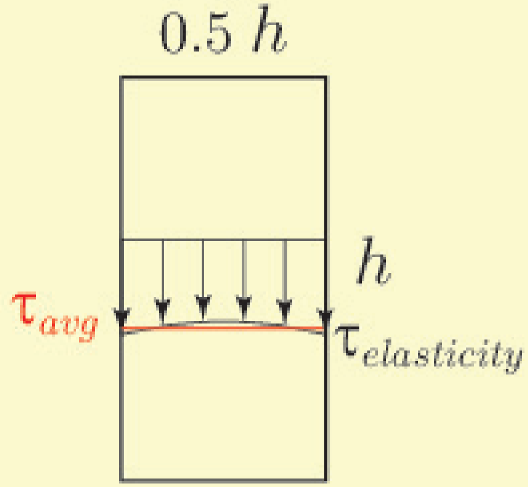
$$\tau_{avg} = \frac{q}{t} = \frac{V_y Q_z}{I_{zz} t}$$

Shear stress - remarks

- **Limitations in derivation:**
  - The derivation of the shear stress formula assumes the shear stress is uniform across the cut line in the cross section - it is essentially the average shear stress.
  - The formula is reasonable in many cases but less so when:
    - The height to width ratio is small
    - The member is not prismatic
    - The edges of the cross section are not parallel to the loading axis
  - For example $\Rightarrow$

Shear stress - remarks

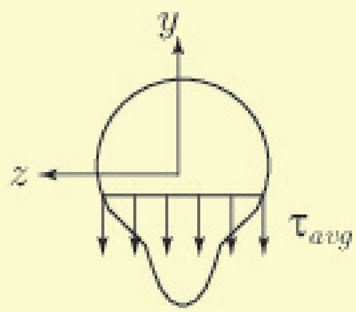
- The actual distribution found from a full three dimensional elasticity solution for a rectangular beam with a height to width ratio of 2. The elasticity solution, $\tau_{elasticity}$ is only about 3% greater than the shear stress τ_{avg}. This difference becomes smaller as the height to width ratio increases.



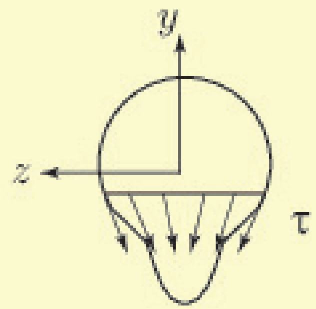
Beams: Shear Stresses

Shear stress - remarks

- Arbitrary cross section with the assumed shear stress profile



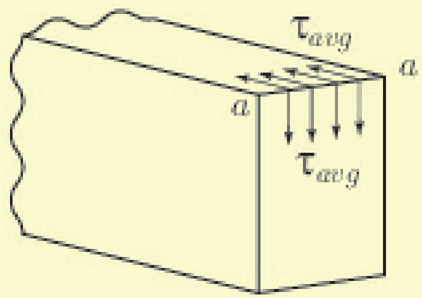
  whereas the real distribution is



- One can use τ_{avg} at points on the cross section that intersect the boundary parallel to the y axis (loading axis)

Shear stress - remarks

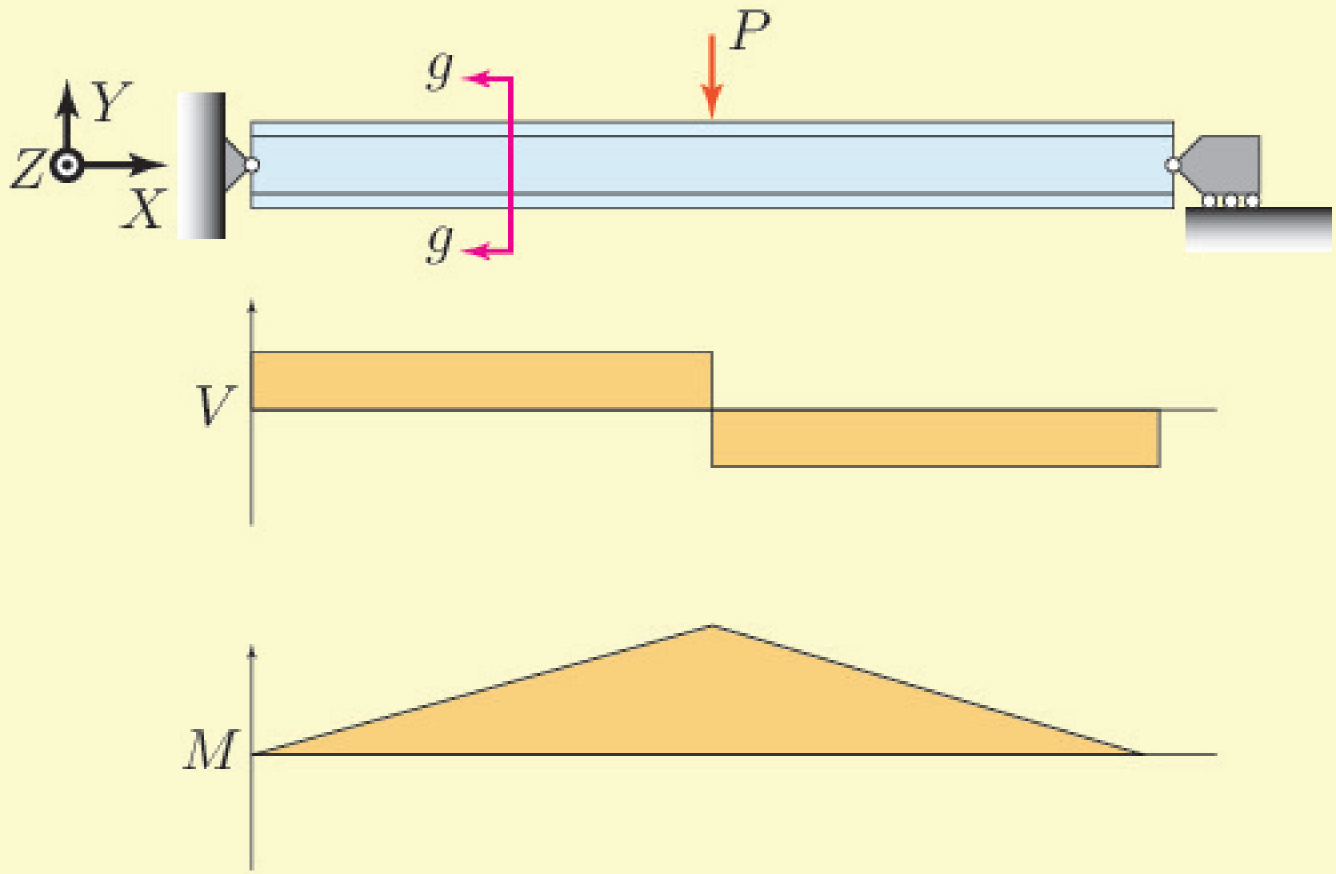
- Since the average horizontal shear stress is know the average transverse shear stress is also known, since shear stresses are equal on mutually orthogonal planes (e.g. $\sigma_{xy} = \sigma_{yx}$)



- In general τ_{avg} will not be a maximum when Q is a maximum (*i.e.* $y = 0$), since τ_{avg} is also dependent on the thickness of the part, t.

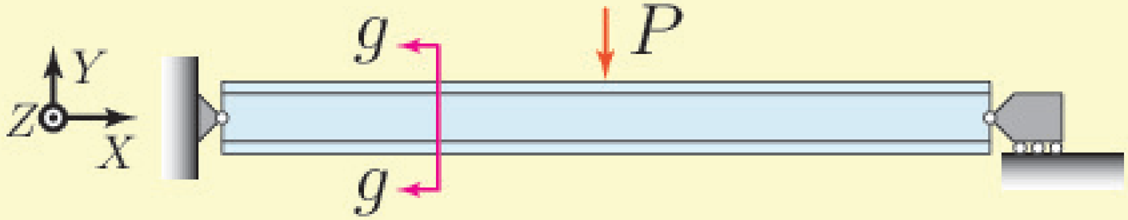
Beams: Shear Stresses

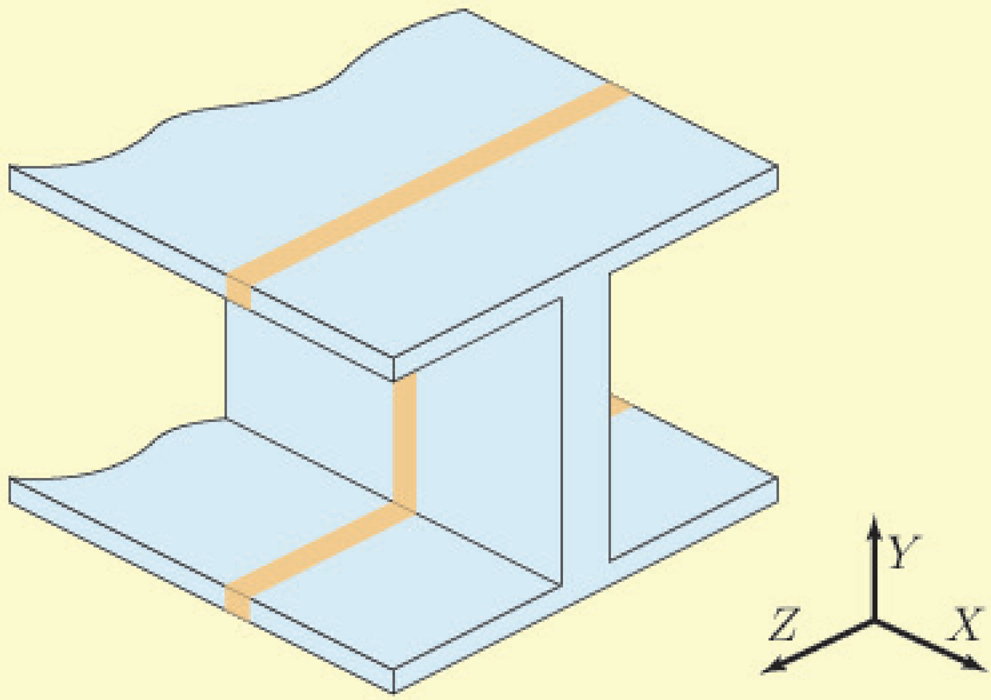
- Visualizing the Shear Stress:
  - **Shear and Moment Diagram:**



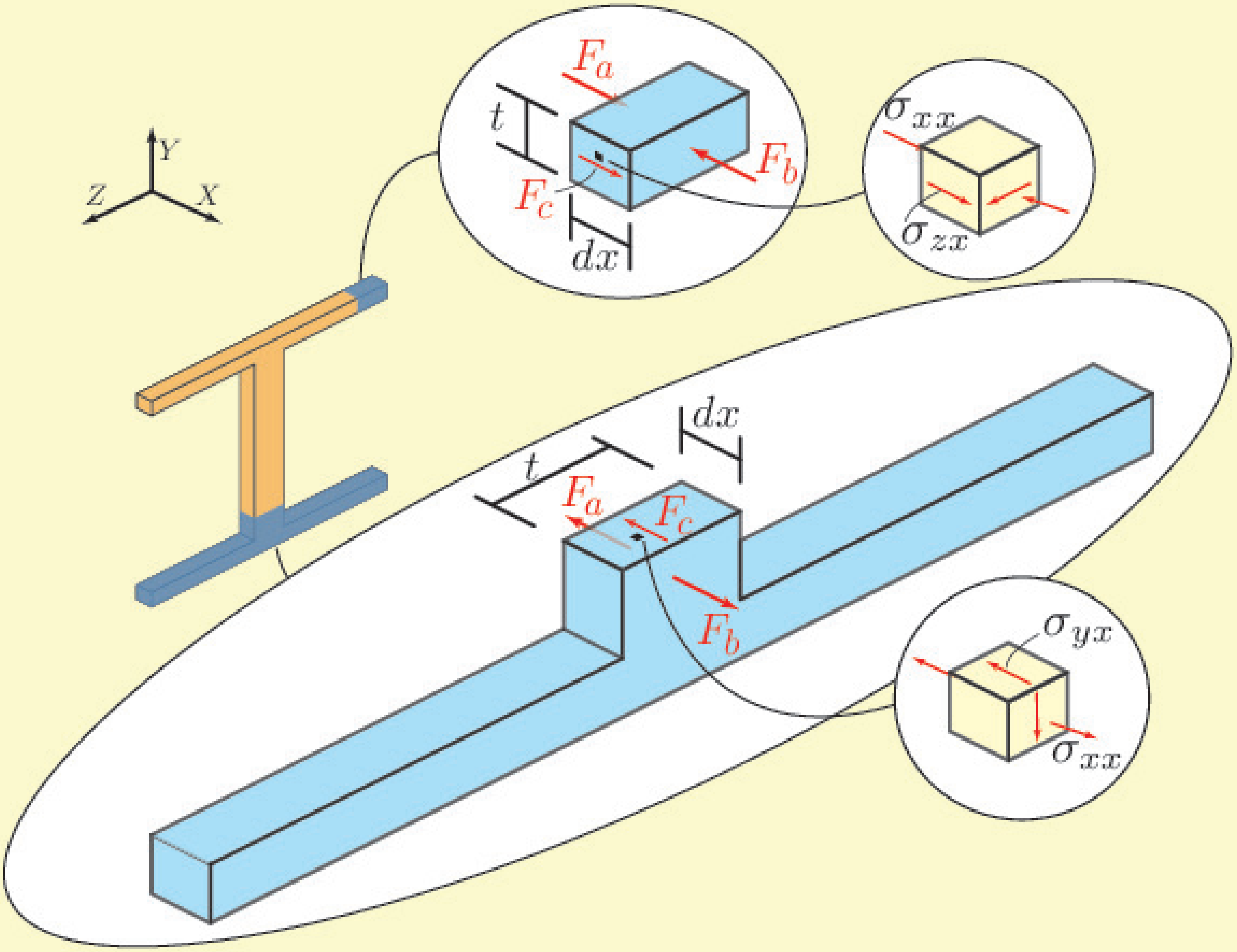
  - **Note the sign convention**

Shear Stress: section cut



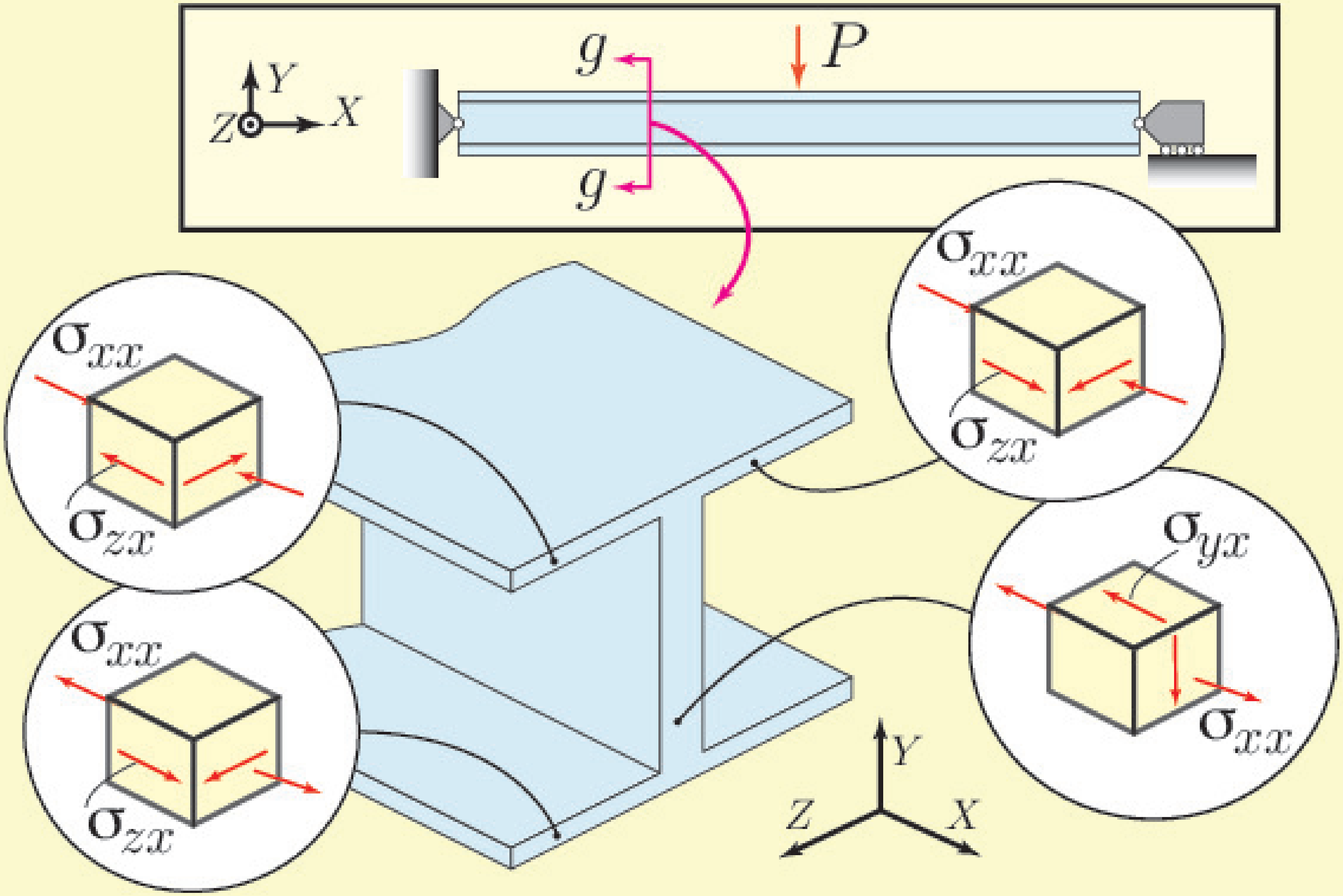


- Shear Stress: consider horizontal forces



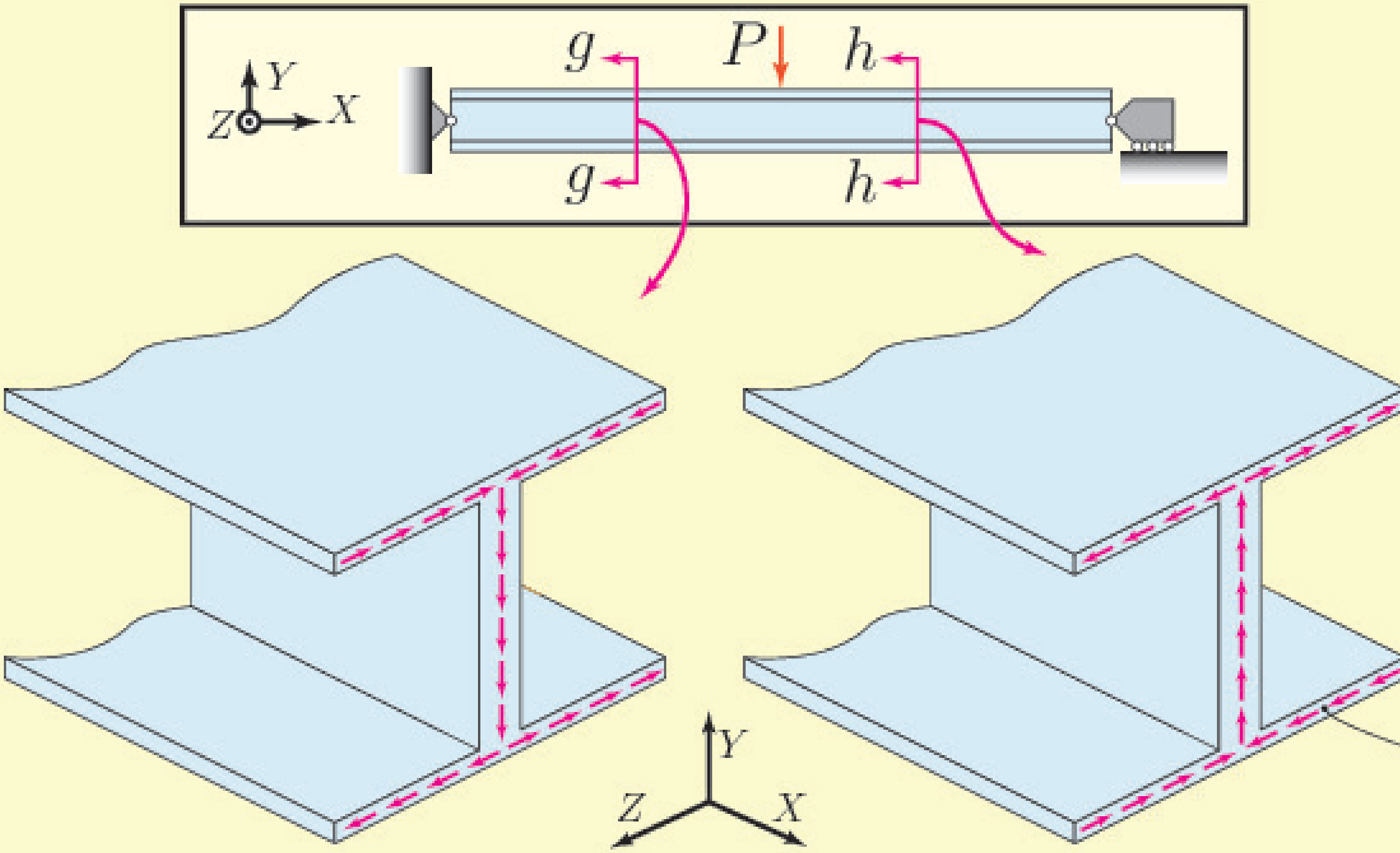
Beams: Shear Stresses

- Shear Stress: orientation of major component

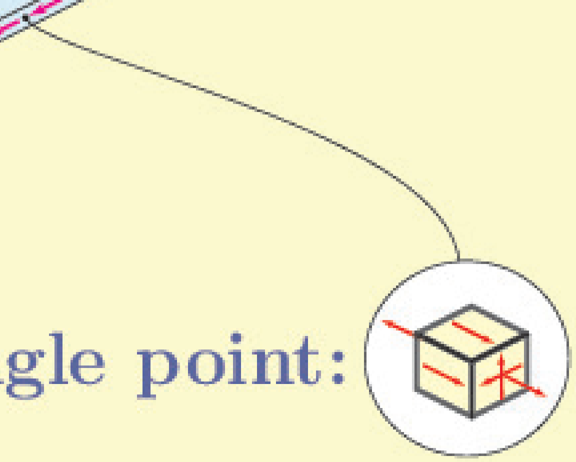


Beams: Shear Stresses

- Shear Stress: major components of shear flow



- **Moment and shear stresses at a single point:**



Beams: Shear Stresses

Summary

- Load, shear, moment relationships

$$-w = \frac{dV}{dx}, \quad V = \frac{dM}{dx}$$

- Moment curvature relation

$$\kappa = \frac{1}{\rho} = \frac{M}{EI}$$

- Flexure formula

$$\sigma = -\frac{My}{I}$$

- Shear flow

$$q = \frac{VQ}{I}$$

- Shear stress

$$\tau = \frac{VQ}{It}$$