An Introduction to Mechanics of Materials

Mechanics I

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An Introduction to Mechanics of Materials

Stress Concentrations

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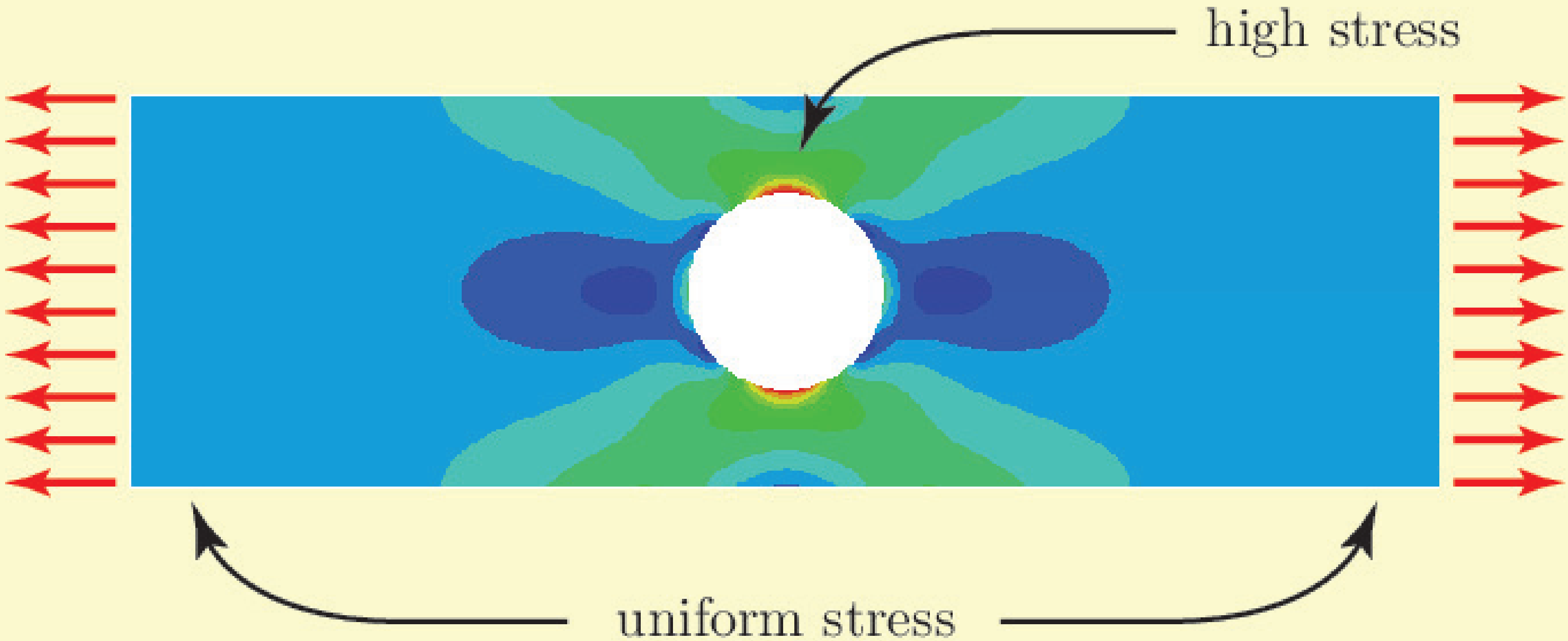
Stress Concentrations

Axial Stress Concentrations

- **High stress gradients exists when there are**
  - Localized boundary/loading conditions
  - abrupt changes in geometry
  - holes/cutouts in geometry
- **Basic stress relations do not apply,** $\sigma \neq \frac{P}{A}$
- **Example** $\rightarrow$

Axial Stress Concentrations

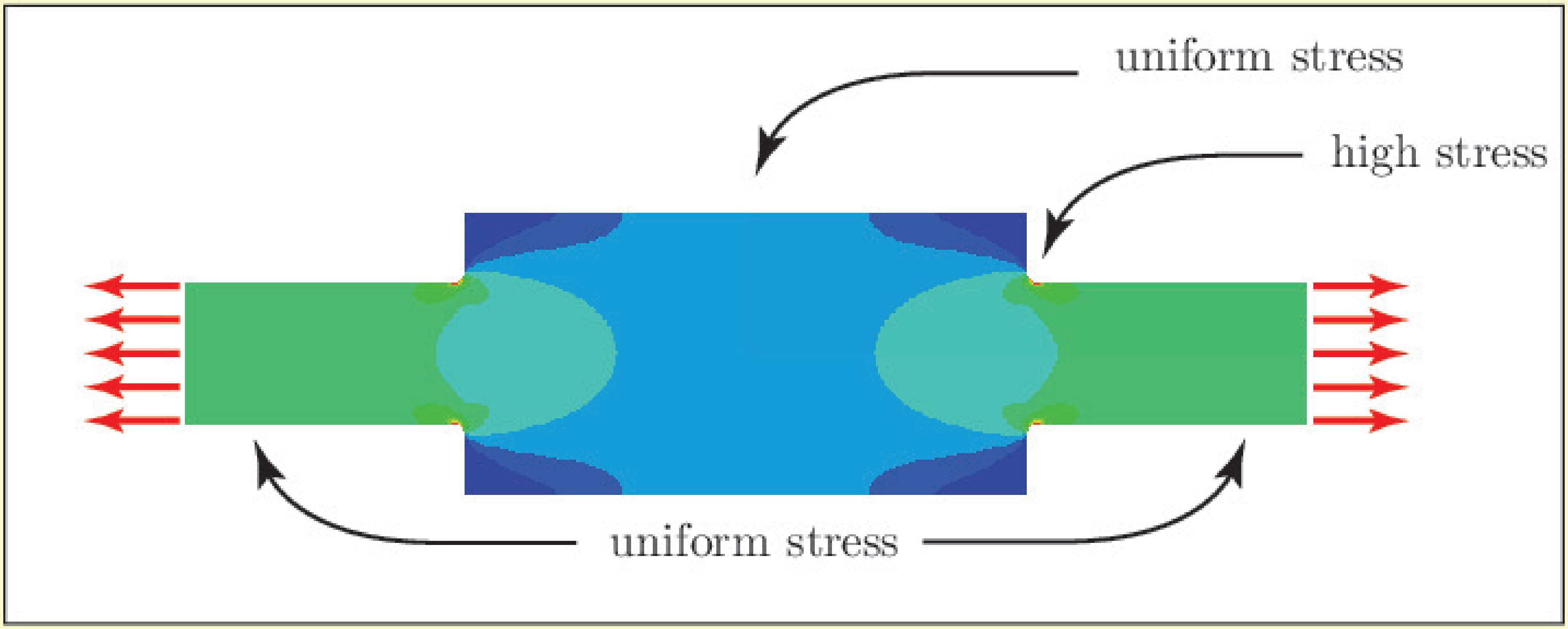
- **Example - strip plate with hole**



Stress Concentrations

Axial Stress Concentrations

- **Example - stepped plate**



Stress Concentrations

Axial Stress Concentrations

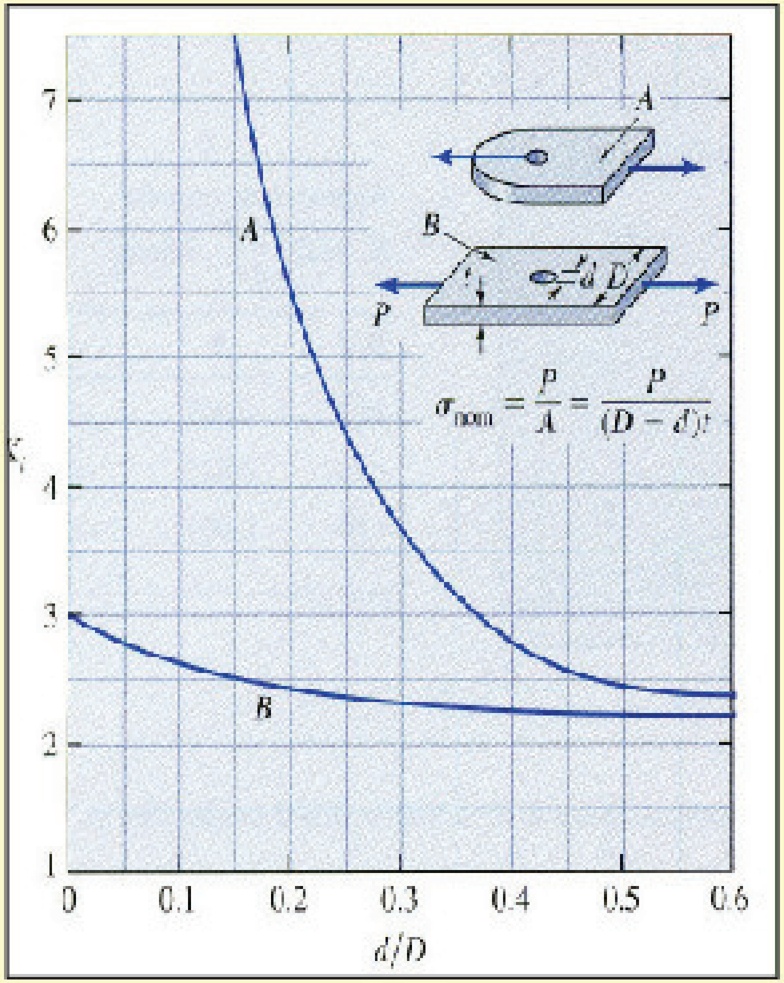
- **Maximum stress can be determined via stress concentration factor, K,**

$$\sigma_{max} = K\sigma_{avg}$$

- **K can be found experimentally or analytically**
- **High stress gradients are very important for brittle materials due to the nature of the material**
- **In ductile materials large stress gradients in local regions are important in**
  - fatigue calculations
  - localization/plasticity problems

Axial Stress Concentrations

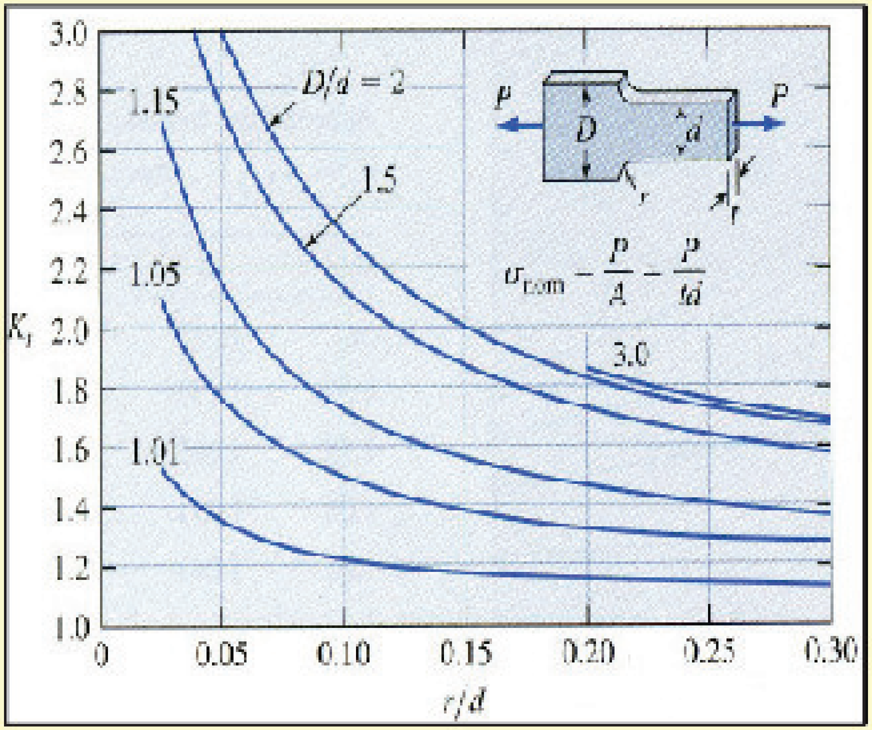
- **Monograph for strip plate with hole**



Stress Concentrations

Axial Stress Concentrations

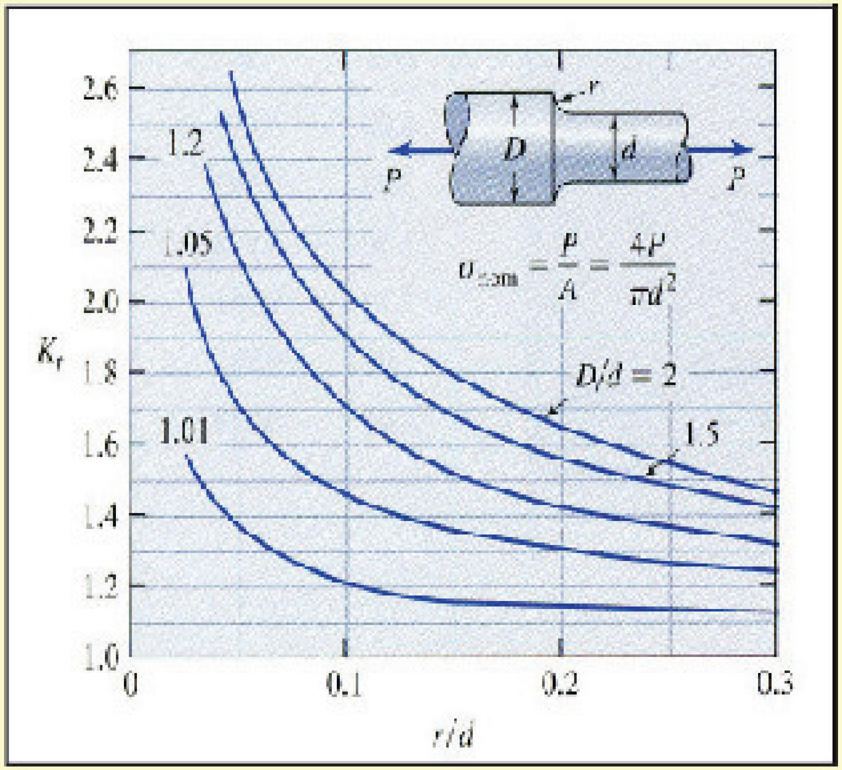
- **Monograph for stepped strip plate**



Stress Concentrations

Axial Stress Concentrations

- **Monograph for stepped circular bar**



Stress Concentrations

Torsion Stress Concentrations

- **High stress gradients exists when there are**
  - boundary or loading conditions
  - abrupt changes in geometry
  - holes/cutouts in geometry

- **Basic stress relations do not apply,** $\tau \neq \frac{Tr}{J}$

Stress Concentrations

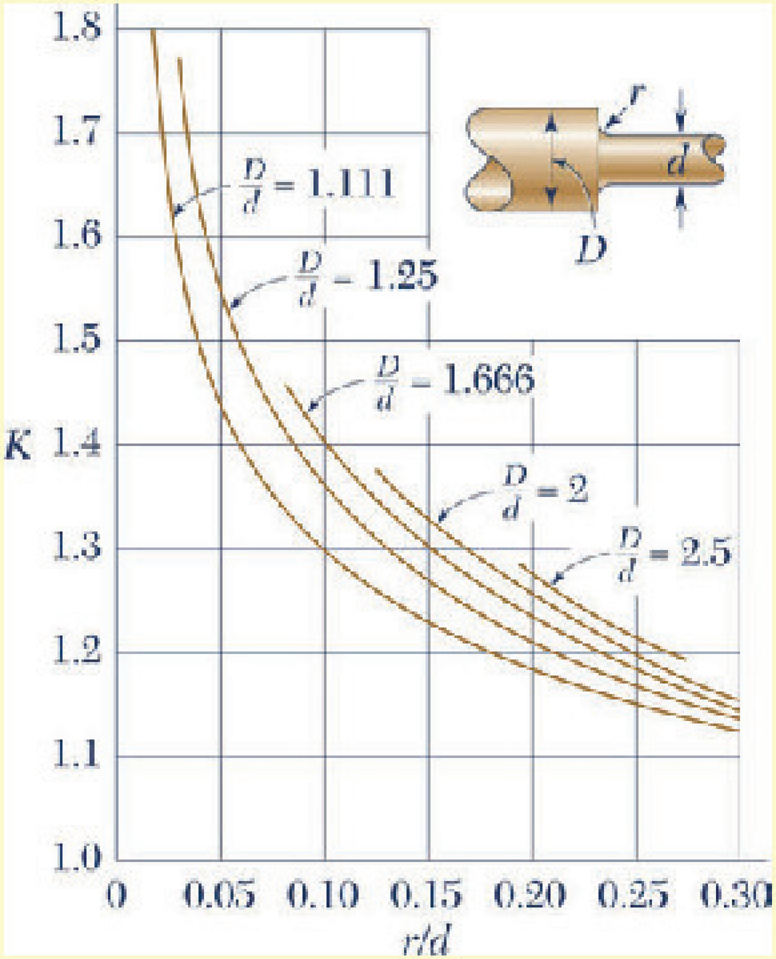
Torsion Stress Concentrations

- **Maximum stress can be determined via stress concentration factor, K,**

$$\tau_{max} = K\tau_{avg}$$

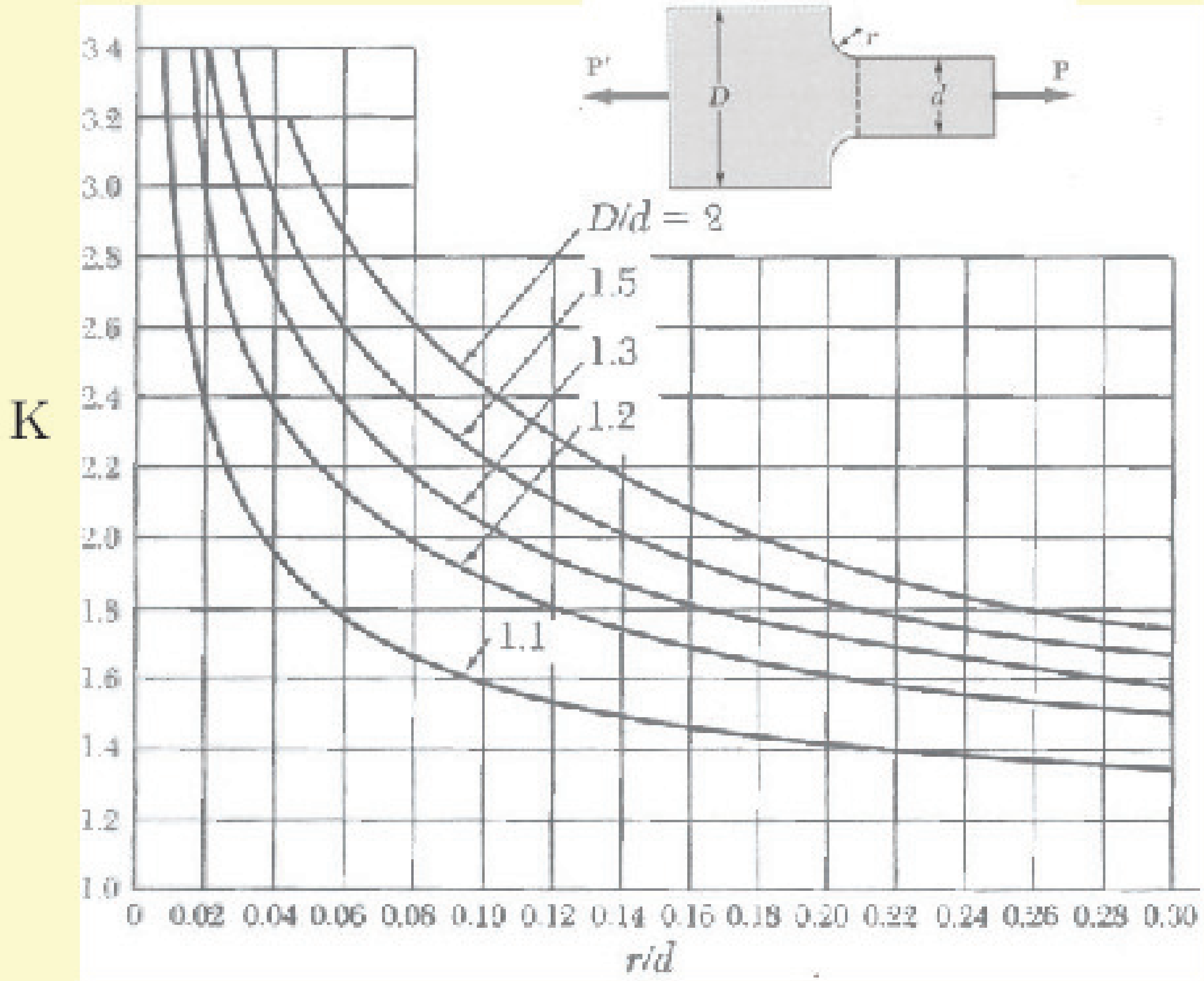
- **K can be found experimentally or analytically**
- **High stress gradients are very important for brittle materials due to the nature of the material**
- **In ductile materials large stress gradients in local regions are important in**
  - fatigue calculations
  - localization/plasticity problems

- Torsion Stress Concentrations
  - **Monograph for stepped shaft**



Stress Concentration Example: #1

- **Given:**



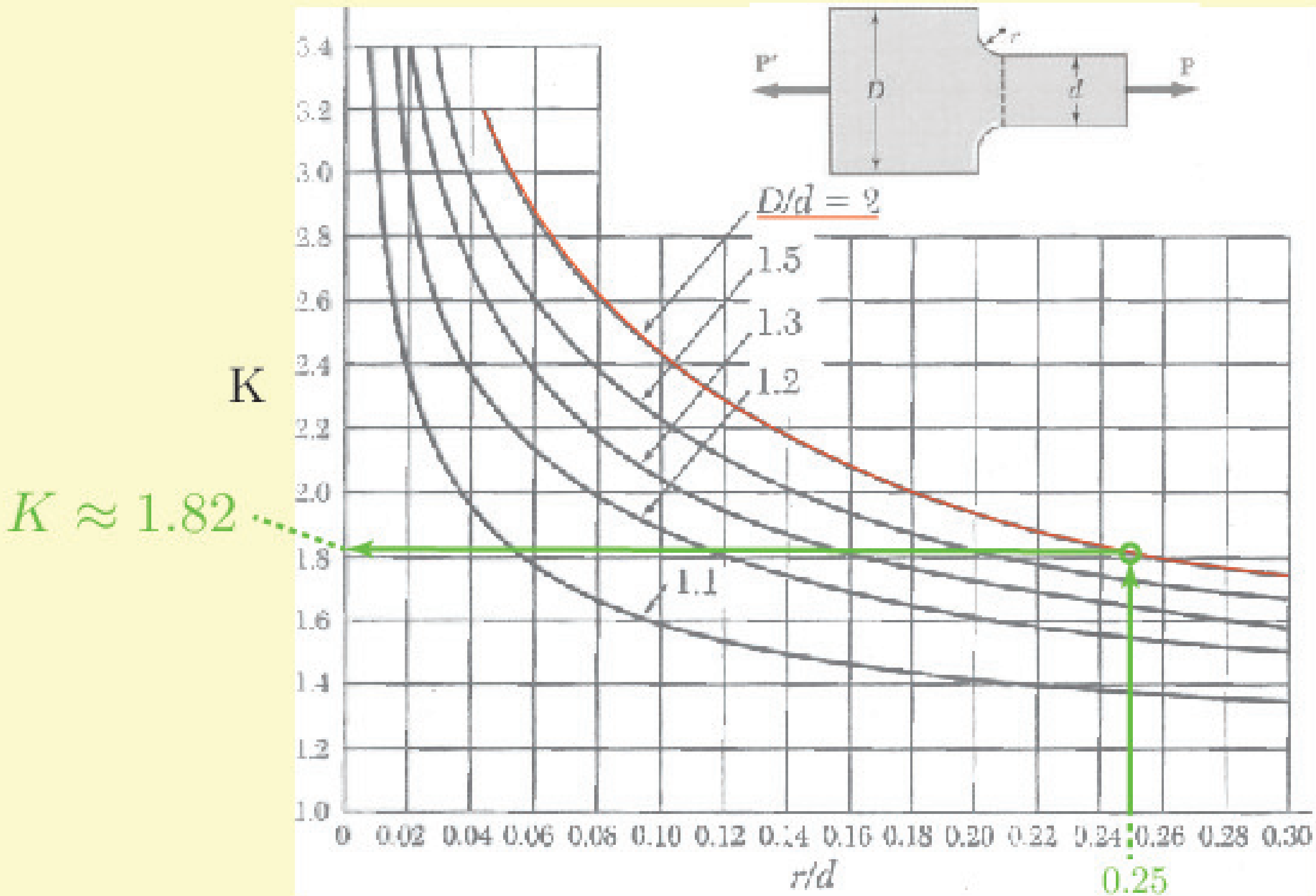
$D = 2\ in$, $d = 1\ in$, $r = 0.25\ in$, $t = 0.5\ in$ (thickness), and $\sigma_{all} = 16.2\ ksi$

- **Find:** The maximum allowable value for P.

Stress Concentration Example: #1

○ Solution:

- Determine the stress concentration factor, K, given $r/d = 0.25$ and $D/d = 2.00$



Stress Concentration Example: #1

Solution:

- The stress concentration factor is $K \approx 1.82$

- Recall stress concentration factor is defined as:

$$K\sigma_{avg} = \sigma_{max}$$

rearrange and set $\sigma_{max} = \sigma_{all} = 16.2\ ksi$ to get:

$$\sigma_{avg} = \frac{\sigma_{max}}{K} = \frac{16.2}{1.82} = 8.9ksi$$

- Compute P

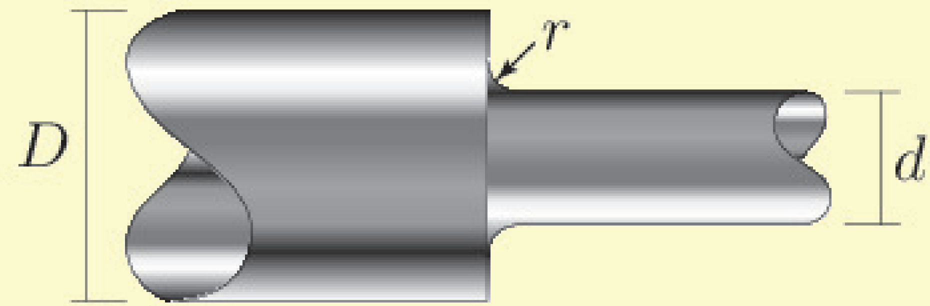
$$P = \sigma_{avg}A = (8.9)(1.0 \times 0.5) = 4.45kips$$

$$\boxed{P = 4.45\ k}$$

Stress Concentration Example: #2

- **Given:** Consider the circular stepped shaft below.

Power, P	$150\ kW$	τ_{all}	$55\ MPa$
d	$75\ mm$	D	$90\ mm$
r	$6\ mm$		



- **Find:** The smallest frequency, f, that the system can undergo. Note that chart on the following page is normalized such that

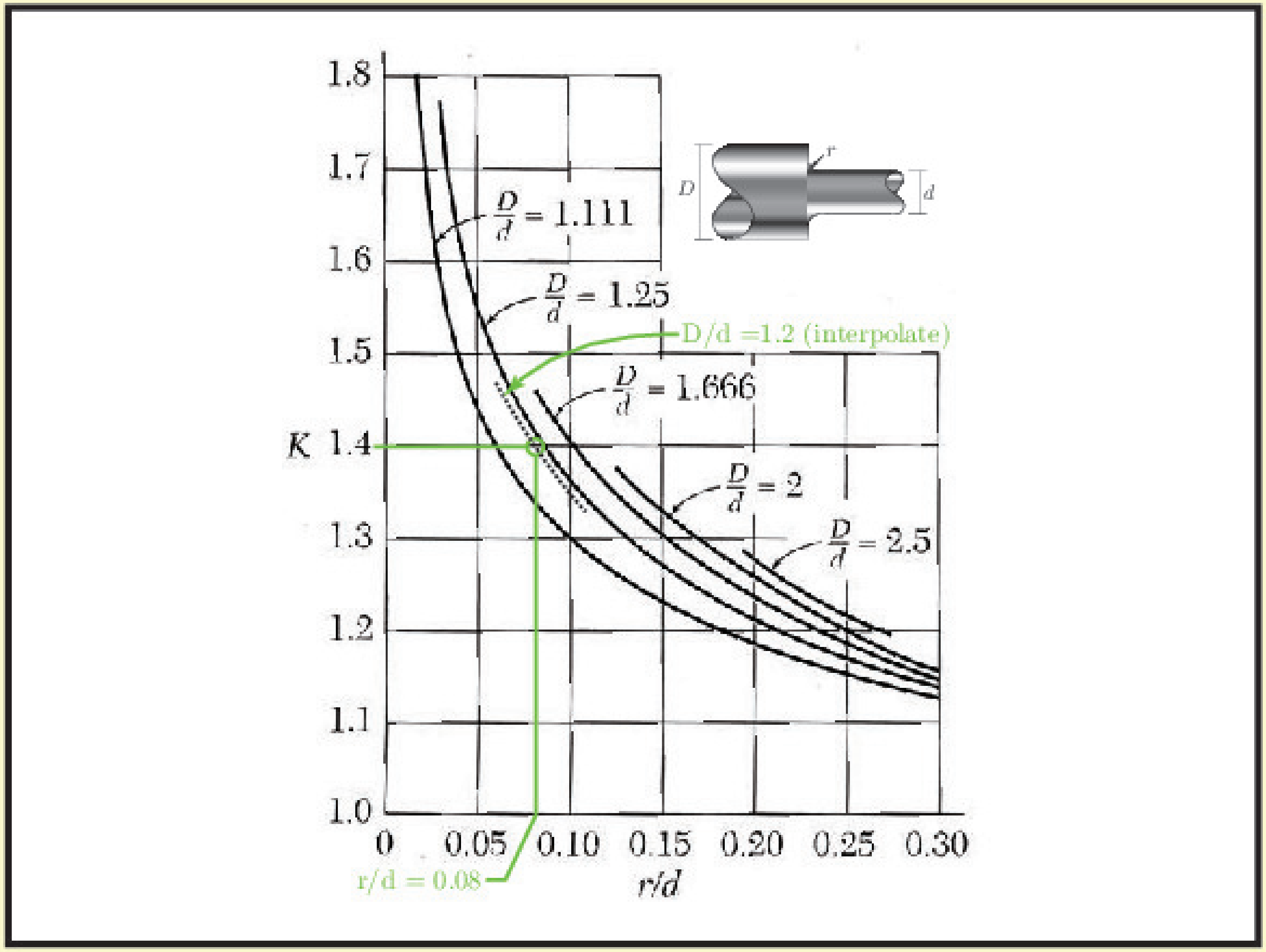
$$\tau_{all} = K\tau_{avg}$$

where

$$\tau_{avg} = \frac{T\bar{r}}{\bar{J}} \quad \text{with} \quad \bar{r} = d/2 \quad \text{and} \quad \bar{J} = \frac{\pi}{2}\bar{r}^4$$

Stress Concentration Example: #2

- **Given:** Monograph for a circular stepped shaft



Stress Concentration Example: #2

○ Solution:

- Determine the stress concentration factor, K, given $r/d = 0.08$ and $D/d = 1.20$ $\rightarrow K \approx 1.40$.

- Find the allowable torque

$$\tau_{all} = K\tau_{avg} = K\frac{T\bar{r}}{\bar{J}} \rightarrow T = \frac{\tau_{all}\bar{J}}{K\bar{r}} = 3.25 \text{ kN - m}$$

- Solve for the frequency, f

$$P = 2\pi f T \ \rightarrow \ f = \frac{P}{2\pi T} = 7.33\ Hz = 440RPM$$