An Introduction to Mechanics of Materials

Mechanics I

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CAL POLY

An Introduction to Mechanics of Materials

Beams: Shear and Moment Diagrams

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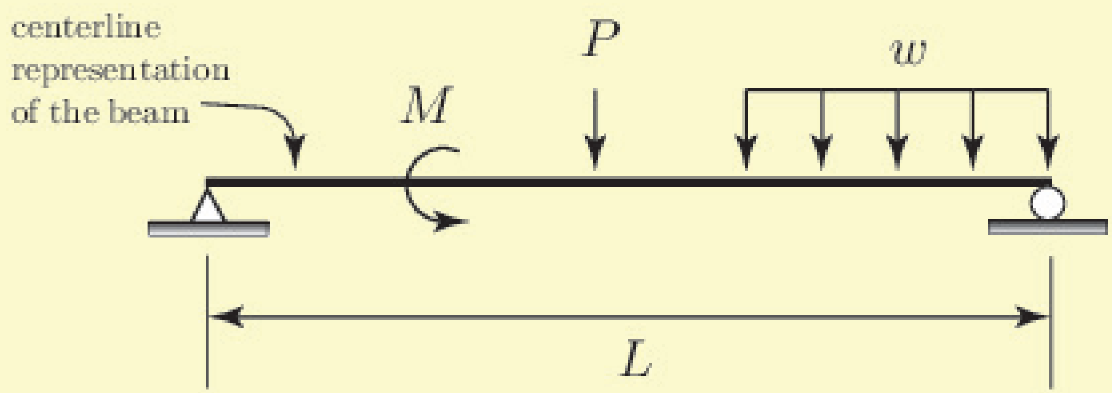
Beams: Shear and Moment Diagrams

Definition:

- Consider a prismatic rod subjected both transverse loads and moments

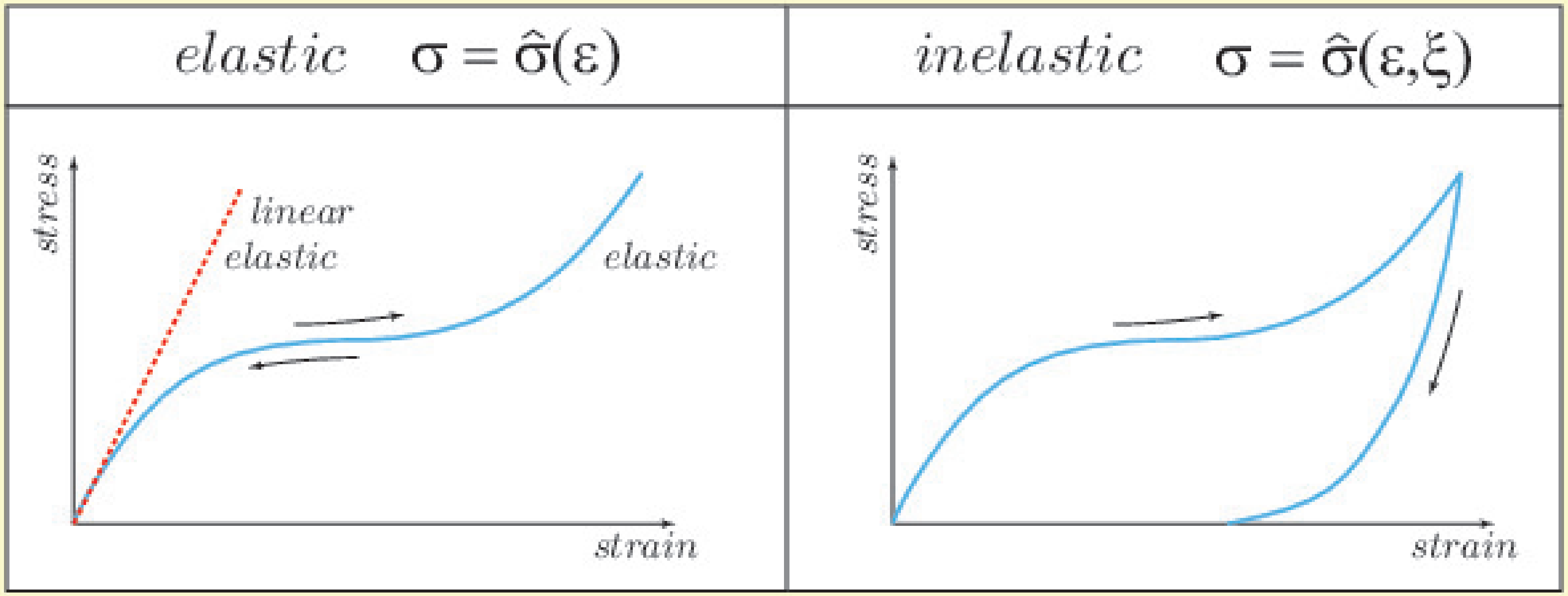


- The predominate dimension $\equiv$ longitudinal axis
- Remaining two directions define the cross-section

Beams: Shear and Moment Diagrams

Assumptions:

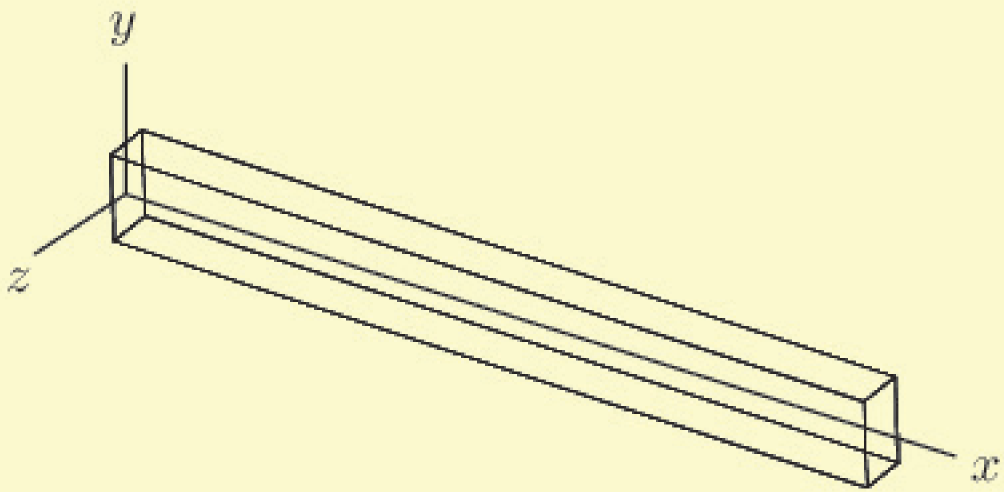
- **The constitutive relationship is linear elastic**

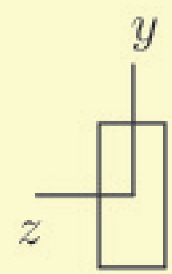


Beams: Shear and Moment Diagrams

- Assumptions:
  - **Displacements/angles and strains are small**

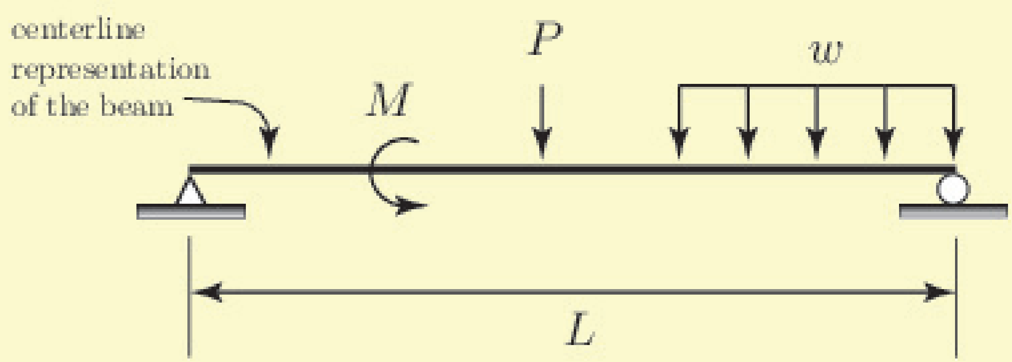
- Assumptions:
  - **Members are straight and prismatic**





Beams: Shear and Moment Diagrams

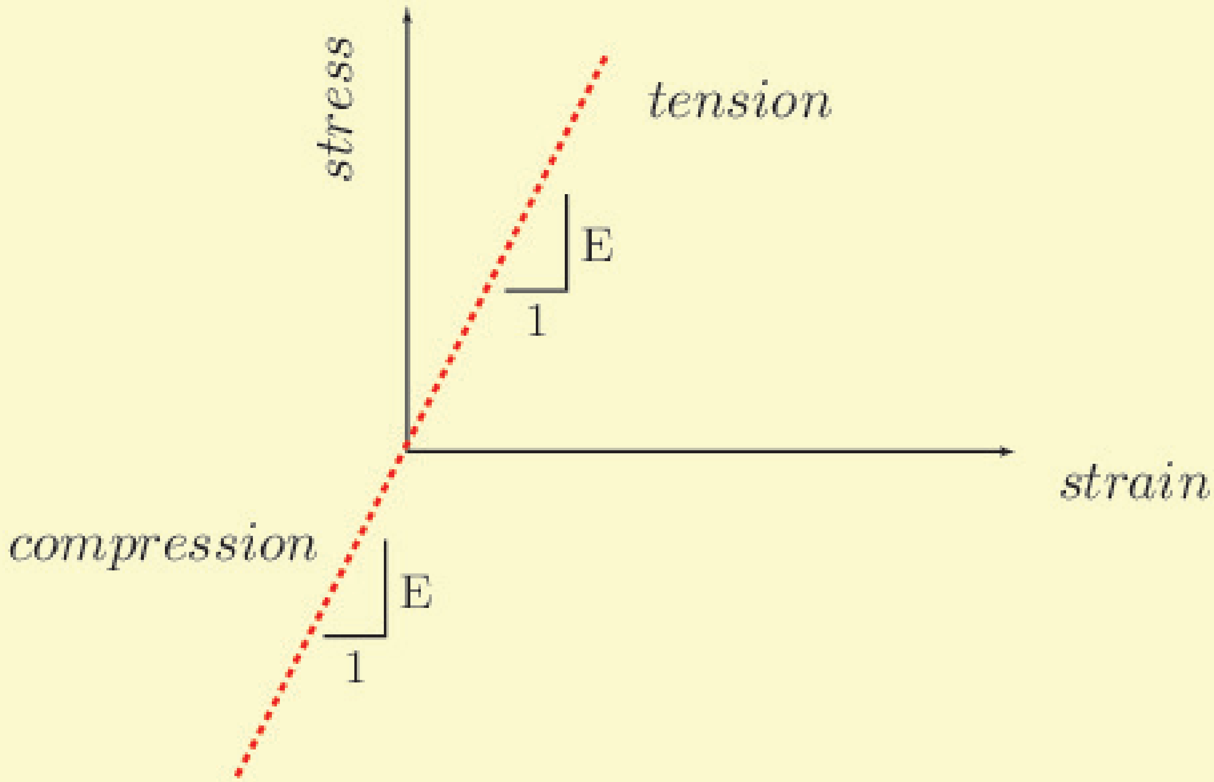
- Assumptions:
  - Moments, either discrete or distributed, are directed normal to the longitudinal axis of the member only, *i.e.* in the $z - dir$
  - Transverse load, either discrete or distributed, are directed normal to the longitudinal axis of the member only, *i.e.* in the $y - dir$



Beams: Shear and Moment Diagrams

- Assumptions:
  - **Localized effects of loading and/or boundary conditions are insignificant via appeal to Saint-Venant's Principle**

- Assumptions:
  - **Material response is the same in all directions**

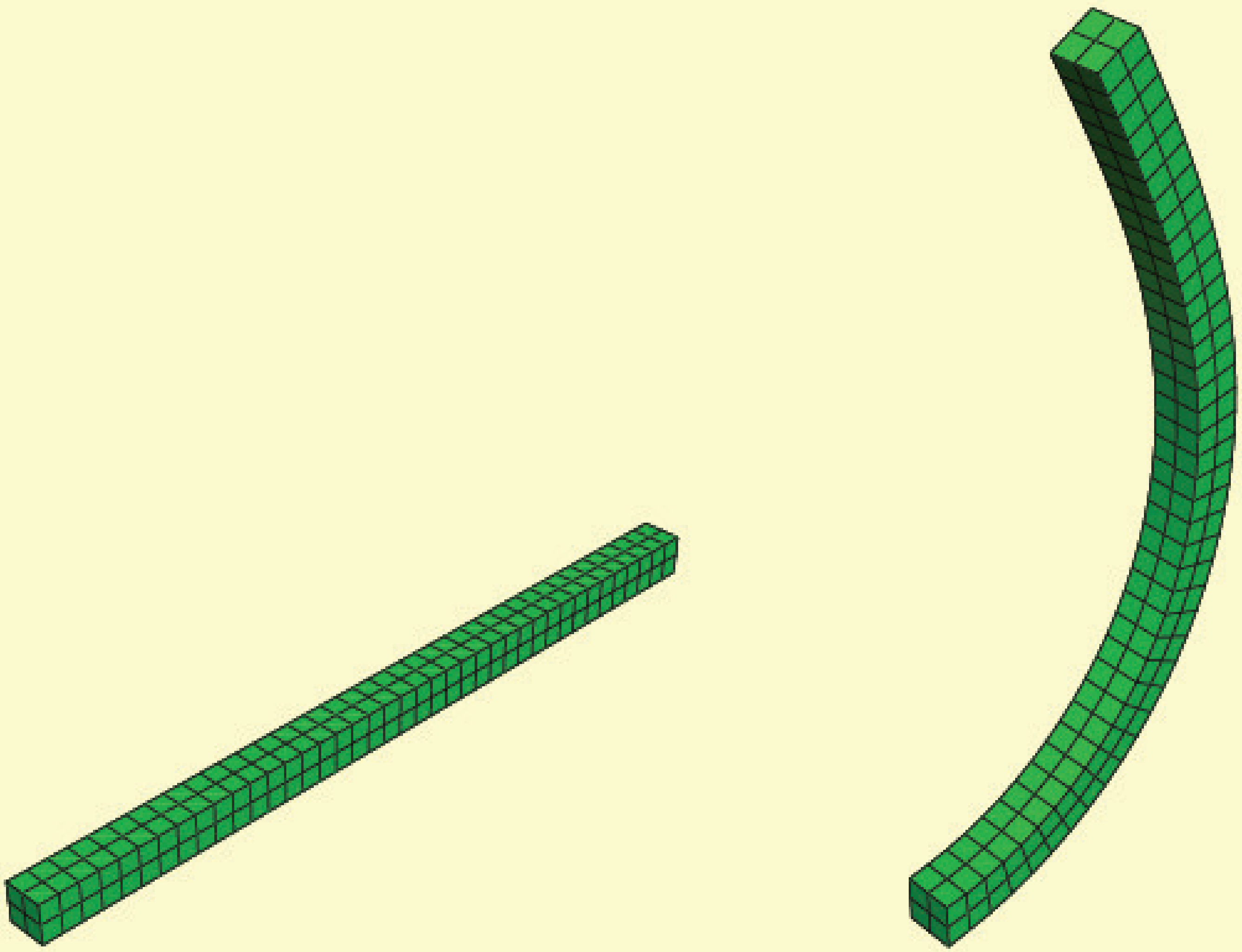


Beams: Shear and Moment Diagrams

- Assumptions:
  - **The effect of either local or global instabilities (*e.g.* buckling) neglected and must be considered separately**

Beams: Shear and Moment Diagrams

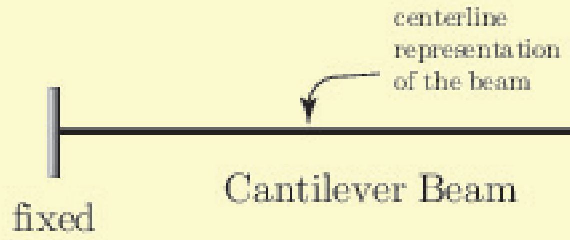
- Motion of the body

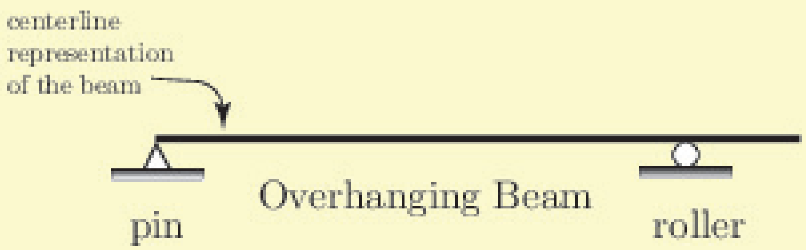


undeformed deformed

Beams: Shear and Moment Diagrams

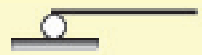
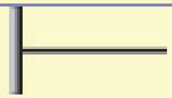
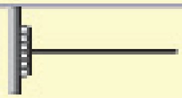
Boundary Conditions





Boundary Conditions

- Common boundary conditions for planar beams

Description	Illustration	u_x	u_y	θ_z
Pin		fixed	fixed	free
Roller		free	fixed	free
Roller		fixed	free	free
Fixed		fixed	fixed	fixed
Moment roller		fixed	free	fixed

Beams: Shear and Moment Diagrams

Determining reactions and member forces

- **Utilizing equilibrium in either two or three dimensions the reactions for an external determinate may be computed.**
- **For a planar two dimensional beam there exists three independent equations of equilibrium:**
  - two translational equations

$$\sum F_x = 0 \, , \quad \sum F_y = 0$$

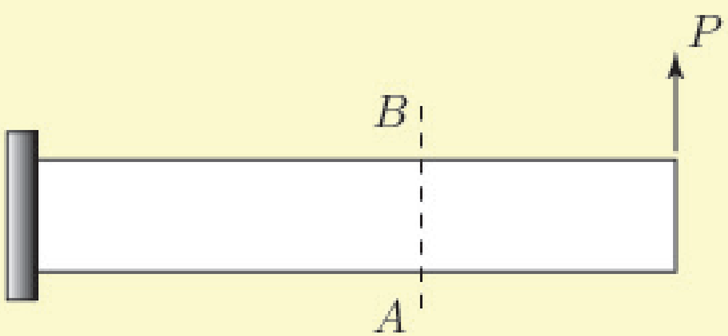
  - one rotation equation

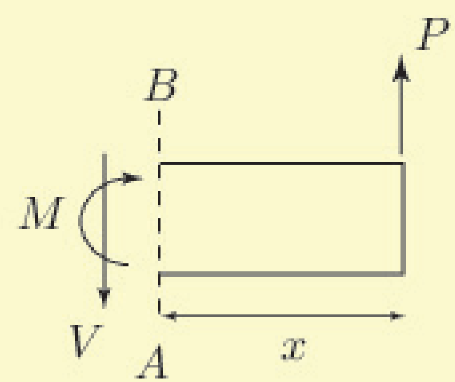
$$\sum M_z = 0$$

Beams: Shear and Moment Diagrams

Shear force and bending moments

- When a beam is loaded internal stresses and strains are created.
- These stresses may be related to forces or moments (also called stress resultants) through integration over the cross section.
- The two stress resultants are a shear force and bending moment.

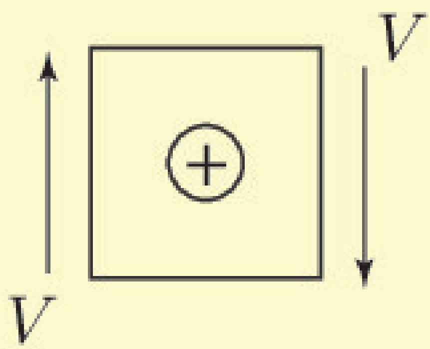


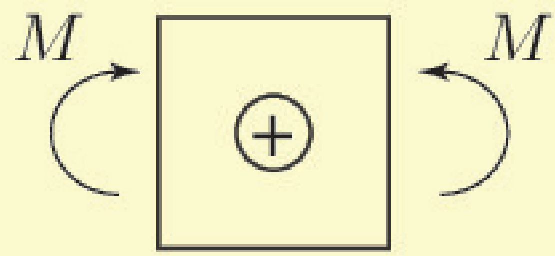


Beams: Shear and Moment Diagrams

Shear force and bending moments: sign convention

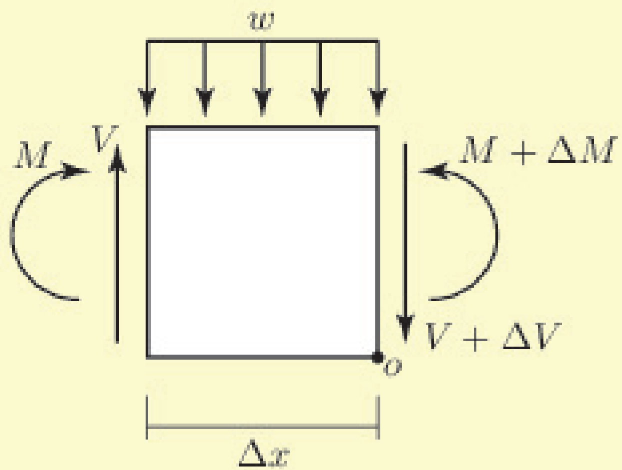
- The sign conventions for the internal stress resultants are developed from what is called the deformation sign convention, which may result in different conventions from the global equilibrium equations.





Beams: Shear and Moment Diagrams

- Relationship between load, shear, and moment
  - Consider an arbitrary slice of a beam



  - Sum forces in the y-direction

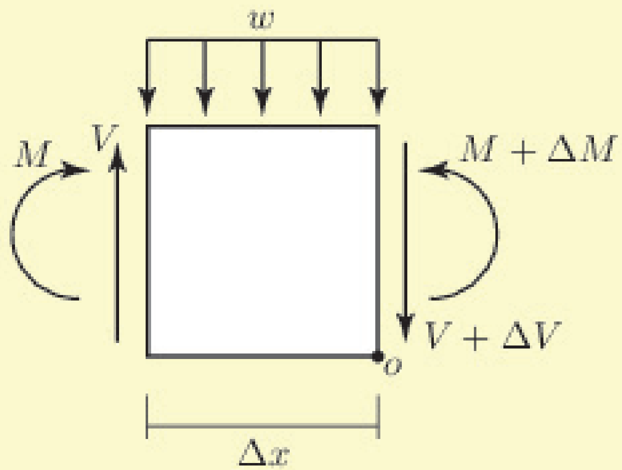
$$\sum F_y = V - (V + \Delta V) - w\Delta x = 0$$

divide by Δx and taking the limit as $\Delta x \to 0$ gives

$$\boxed{\frac{dV}{dx} = -w}$$

Beams: Shear and Moment Diagrams

- Relationship between load, shear, and moment
  - Consider an arbitrary slice of a beam



  - Consider moments about the point "o"

$$\sum M_o = (M + \Delta M) - M - V\Delta x + \frac{1}{2}w\Delta x^2 = 0$$

divide by Δx and taking the limit as $\Delta x \to 0$ gives

$$\boxed{\frac{dM}{dx} = V}$$

Beams: Shear and Moment Diagrams

- Relationship between load, shear, and moment
  - **Remarks**
    - The slope of the shear diagram is the negative of the magnitude of the distributed load w
    - The area under the loading diagram is related to the shear diagram

$$\frac{dV}{dx} = -w \quad \Rightarrow \quad dV = -w dx \quad \Rightarrow \quad \int_a^b dV = -\int_a^b w dx \quad \Rightarrow$$

$$V_b - V_a = -\int_a^b w dx \quad \Rightarrow \quad V_b = V_a - \int_a^b w dx$$

Relationship between load, shear, and moment

- **Remarks**
  - The slope of the moment diagram is equal to the magnitude of the shear, V
  - The area under the shear diagram is related to the moment diagram

$$\frac{dM}{dx} = V \quad \Rightarrow \quad dM = V\,dx \quad \Rightarrow \quad \int_a^b dM = \int_a^b V\,dx \quad \Rightarrow$$

$$M_b - M_a = \int_a^b V\,dx \quad \Rightarrow \quad M_b = M_a + \int_a^b V\,dx$$

Beams: Shear and Moment Diagrams

- Shear force and bending moment diagrams
  - For design the analyst needs to know the distribution of the internal forces to determine the maximum values
  - Knowing this distribution allows for "optimization" of the beam with respect to material and geometry
  - A convenient way to provide this information is to draw a graph showing how the internal shear, V and the internal moment, M vary with the position of the beam, x